The Definition of a Logarithm

Assume that b, M, and y are real numbers, $M > 0$, $b > 0$, and $b \neq 1$.

Then $\log_b M = y$ means $b^y = M$.

(In other words, $\log_b M$ tells you to ask yourself, "b to what power gives me M?")

Example: Change the equation $\log_3 a = n + 5$ to its equivalent exponential form.

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Example: Change the equation $5^{p+1} = q - 3$ to its equivalent logarithmic form.

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Example: Simplify $\log_2 32$.

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Example: Simplify $\log_4 \frac{1}{16}$.

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Example: Simplify $\log_5 (-5)$.

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Example: Solve for x in the equation $\log_2 (x + 1) = 3$.

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Example: Solve for b in the equation $\log_b 8 = \frac{3}{5}$.

More on Simplifying Expressions Involving Logarithms

Assume that b, M, and y are real numbers, $M > 0$, $b > 0$, and $b \neq 1$.

Then $b^{\log_b M} = M$.

Proof:

Let $b > 0$ (but not equal to 1), and let $M > 0$.

Let $\log_b M = x$.

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Example: Simplify the expression $4^{\log_4 (x-5)}$. (Assume that $x > 5$.)

Another Property of Logarithms

Assume that b, M, and N are real numbers, $M > 0$, $N > 0$, $b > 0$, and $b \neq 1$. Then $M = N$ if and only if $\log_b M = \log_b N$.

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Properties of Logarithms

Assume that b, M, N, and p are real numbers, $M > 0$, $N > 0$, $b > 0$, and $b \neq 1$. Then:

$\log_b (MN) = \log_b M + \log_b N$ (This is called the *Product Rule*.)

$\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$ (This is called the *Quotient Rule*.)

$\log_b (M^p) = p \log_b M$ (This is called the *Power Rule*.)

Proof of the Product Rule:

Let $b > 0$, and let x and y represent real numbers.

Let $b^x = M$, and let $b^y = N$.

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Proof of the Power Rule:

Let $b > 0$, and let p represent a real number.

Let $b^x = M$. (So $M > 0$ and $x = \log_b M$.)

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Example: Expand the expression $\log_5 (7x^2)$ into a sum or difference of simpler logarithms.

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Example: Condense the expression $7\log_6 a - \log_6 b + 4\log_6 (c - 1)$ into a single logarithm and simplify the result.

The Definitions of the Common Logarithm and the Natural Logarithm

If the base of a logarithm is not stated, we assume it's 10.

$\ln M$ is defined to mean $\log_e M$

Example: Solve for x in the equation $\log (x - 3) = 4$.

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Example: Simplify $\log 10^{14}$.

Method #1: Use the fact that $\log_b M$ tells you to ask yourself, "b to what power gives me M?")

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Method #2: Use the fact that $\log_b (M^p) = p \log_b M$.

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Example: Simplify $\ln e^{x+2}$.

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Example: Expand the expression $\log \sqrt{\frac{10x^3}{y}}$ into a sum or difference of simpler logarithms and simplify the result.

The Change-of-Base Formula

$$\log_b M = \frac{\log M}{\log b} = \frac{\ln M}{\ln b}$$

Proof of the Change-of-Base Formula:

Let $b > 0$ (but not equal to 1), and let $M > 0$.

Let $\log_b M = x$.

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Example: Approximate $\log_3 17$ to three decimal places.