What is an *Asymptote*?

Rational functions often have <u>asymptotes</u>, which are lines that the graph is trying to get very close to. Vertical asymptotes will never be crossed, but horizontal asymptotes can be crossed.

Example: Graph the function $f(x) = \frac{1}{x+3}$ in the viewing rectangle [–20, 20] by [–10, 10]. Then fill in the table of values below, and use this to describe the horizontal and vertical asymptotes of the function.

x	y
–50	
–20	
–3.5	
–3.1	
–3.01	
–3	
–2.99	
–2.9	
–2.5	
20	
50	

Why Does a Denominator Equal to Zero Create a Problem?

Introduction to Rational Functions

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Example: Graph the function $g(x) = \frac{-x^2 + 10x + 12}{x^2 - 5x - 6}$ in the viewing rectangle [–20, 20] by [–15, 15]. Then describe the horizontal and vertical asymptotes of the function.

Graphing Rational Equations of the form $f(x) = \frac{P(x)}{Q(x)}$ (where the degree of $P(x)$ is less than or equal to the degree of $Q(x)$ and there are no holes)

- To find the y-intercept of $f(x)$, set x equal to zero (in the original equation) and solve for y.
- The x-intercept(s) will match those of $P(x)$. So, to find the x-intercepts of $f(x)$, set $P(x)$ equal to zero and solve for x.
- The vertical asymptote(s) will be the value(s) of x that make the denominator equal zero.
- To find the horizontal asymptote, you can use one of the following rules:
  (1) If the degree of $P(x)$ is less than the degree of $Q(x)$, there will be a horizontal asymptote at $y = 0$.
  (2) If the degree of $P(x)$ is equal to the degree of $Q(x)$, use the leading coefficients of $P(x)$ and $Q(x)$.
- Vertical asymptotes cannot be crossed, but horizontal asymptotes can be crossed. To find out where (if at all) a graph crosses its horizontal asymptote, divide using either synthetic or long division and set the remainder equal to zero.

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Example: Find the x- and y-intercepts of the function $h(x) = \frac{1 - 4x^2}{3x^2 - 12x - 15}$.

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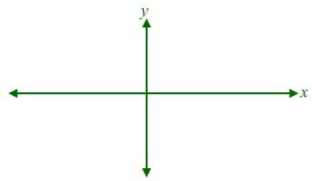
Example: Graph the function $h(x) = \frac{1 - 4x^2}{3x^2 - 12x - 15}$. Label the asymptotes and intercepts.

To figure out where (if at all) the graph crosses the horizontal asymptote of $y = -\frac{4}{3}$:

$$\begin{array}{r} -\frac{4}{3} \\ 3x^2 - 12x - 15 \overline{)\, -4x^2 + 0x + 1} \\ -(-4x^2 + 16x + 20) \\ -16x - 19 \end{array}$$

$-16x - 19 = 0$

$x = -1.1875$



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Example: Find the x- and y-intercepts of the function $k(x) = \frac{5 - 2x}{x^2}$.

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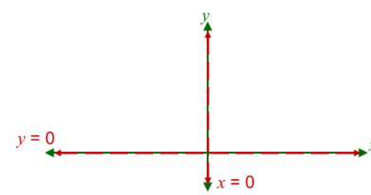

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Example: Find the x- and y-intercepts of the function $r(x) = \frac{9}{x^2 + 4}$.

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Example: Graph the function $r(x) = \frac{9}{x^2 + 4}$. Label the asymptotes and intercepts.