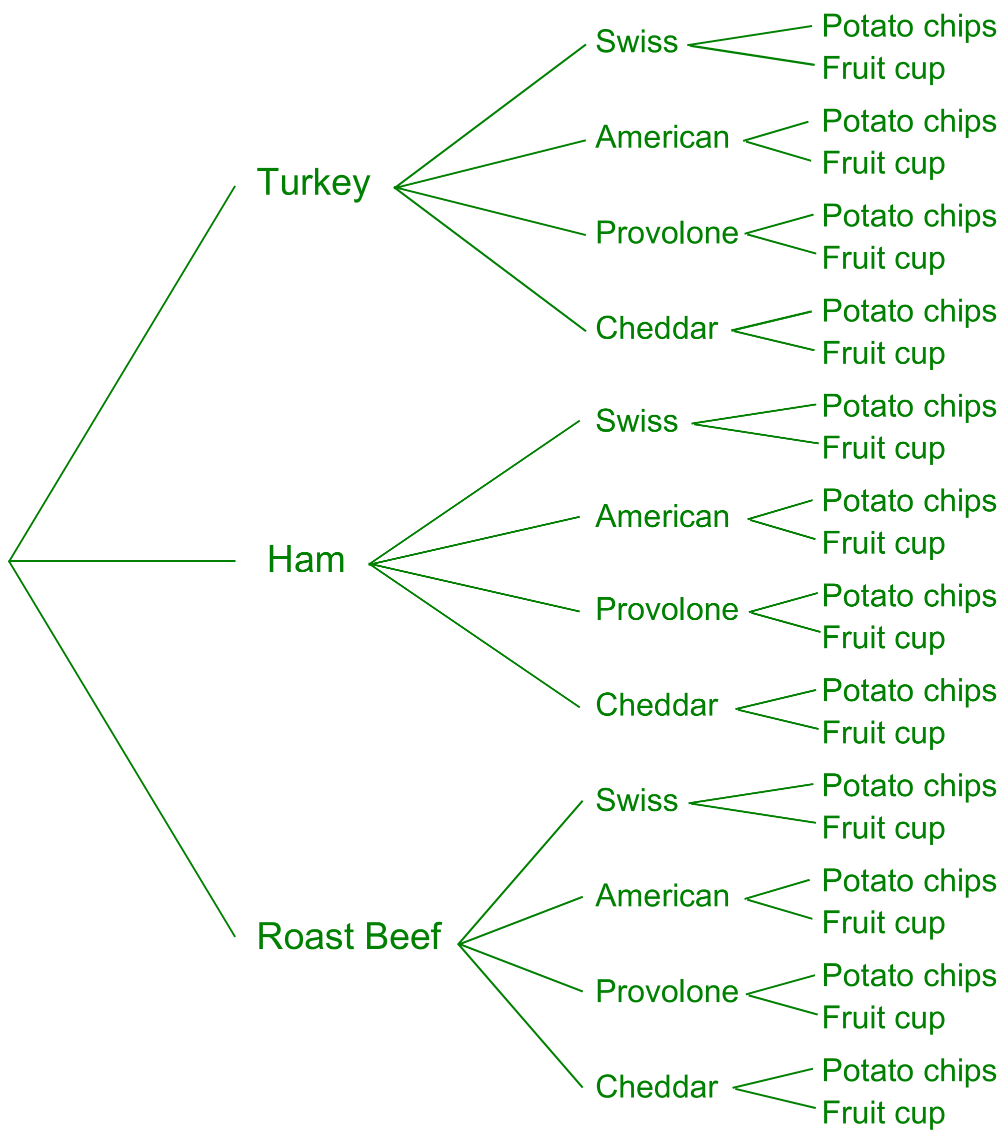
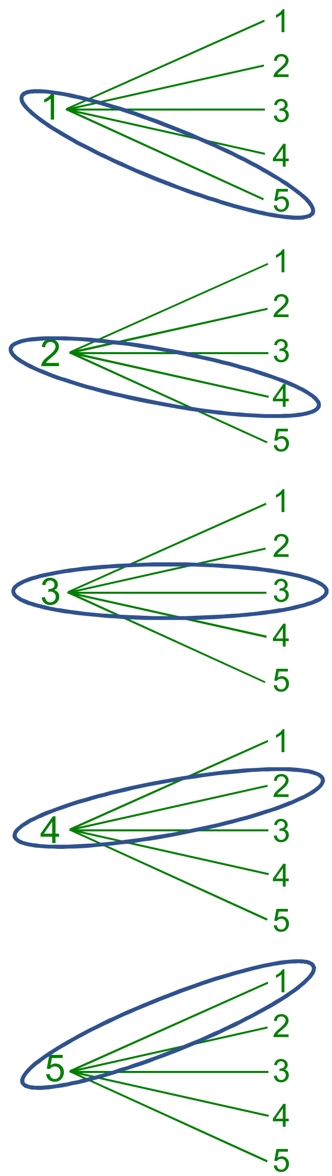
A sandwich shop offers three different kinds of meat (turkey, ham, and roast beef) and four kinds of cheeses (Swiss, American, provolone, and cheddar) on their sandwiches. They also let you choose potato chips or a fruit cup to go with your sandwich. If you would like to get a meat and a cheese on your sandwich along with a side item, describe all the possible orders you can make. How many different orders are possible?



Suppose that the numbers 1 through 5 are placed in a hat. If a number will be drawn and then replaced before a second number is chosen, how many different ways can you draw a sum of 6? Assume that order is important. (In this case, we say that repetition is possible.)

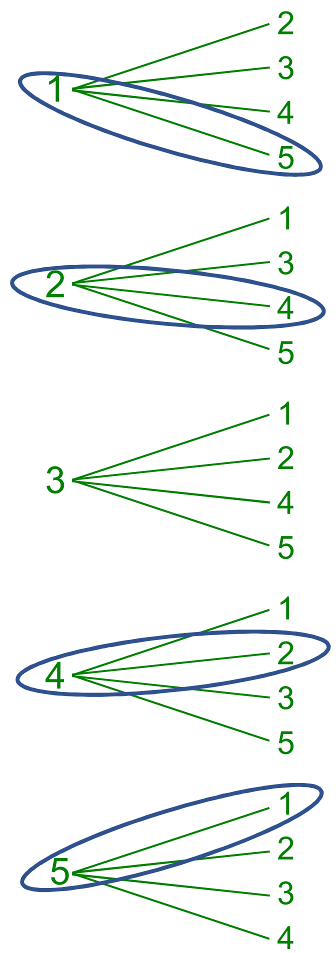
The tree diagram below describes all the possibilities, and the circled ones describe how you can get a sum of 6.



Since there are 5 circles, there are 5 different ways that you can get a sum of 6.

Suppose that the numbers 1 through 5 are placed in a hat. If a number will be drawn and will NOT be replaced before a second number is chosen, how many different ways can you draw a sum of 6? Assume that order is important. (In this case, we say that repetition is not possible.)

The tree diagram below describes all the possibilities, and the circled ones describe how you can get a sum of 6.



Since there are 4 circles, there are 4 different ways that you can get a sum of 6.

Suppose that the numbers 1 through 5 are placed in a hat. If a number will be drawn and will NOT be replaced before a second number is chosen, how many different ways can you draw a sum of 6? Assume that order is not important.

Possibilities: 1 and 2

1 and 3

1 and 4

1 and 5

2 and 3

2 and 4

2 and 5

3 and 4

3 and 5

4 and 5

The Fundamental Counting Principle

Example: You are trying to figure out a 5-letter word in a crossword puzzle. You know all the letters in the word except for the second and third letters. If you know that the second letter is either an E or an O, how many different possibilities are there?

The Fundamental Counting Principle

Example: Suppose that you are the manager of a baseball team. For an upcoming game, you know which nine players you want to start the game, but you can't decide on the order in which they should come up to bat. (a) How many different ways are there of arranging the starting batting order? (b) If you know that you want the pitcher to bat last, how many different ways are there of arranging the starting batting order?

The Fundamental Counting Principle

Example: How many different 7-digit telephone numbers can you make if the first digit cannot be a 1 or a 0?

Factorials

For any integer $n > 0$, $n!$ (read "n factorial") is defined to equal the product of all the positive integers less than or equal to n, and 0! is defined to equal 1.

Example: Evaluate (a) 5! and (b) 8!.

(a) $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$= 120$

(b) $8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$= 40{,}320$

Permutations and Combinations

When a group of people or objects is arranged in a certain order, the arrangement is called a permutation. The number of permutations that are possible with n objects taken r at a time can be found using the formula $P(n, r) = \frac{n!}{(n-r)!}$.

When a group of people or objects is arranged in a way that the order does not matter, the arrangement is called a combination. The number of combinations that are possible with n objects taken r at a time can be found using the formula $C(n, r) = \frac{n!}{r!(n-r)!}$.

Example: A club has 20 members. If the club needs to elect a President, Vice-President, and Secretary from among these members, how many different ways can this be done?

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Example: A club has 20 members. If the club needs to elect a 3-person committee from among these members, how many different ways can this be done?

Permutations and Combinations

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Example: A teacher will give a matching test that consists of 10 questions and 12 different answers. If an answer is used, then it will be used only once. How many different answer sheets are possible?