Chapter 1 – Review of Equations and Inequalities

Part I – Review of Basic Equations

Recall that an equation is an expression with an equal sign in the middle. Also recall that, if a question asks you to solve an equation for a variable, the question wants you to find the value(s) of the variable that will make the equation true. When you are solving equations, you will need to remember the following general principles.

- To solve equations, we use the Addition Property of Equality, the Subtraction Property of Equality, the Multiplication Property of Equality, and the Division Property of Equality. Essentially, these properties say that you can do almost anything to an equation as long as you do the same thing to both sides.
- To cancel out something that is added to, subtracted from, multiplied by, or divided by you variable, you do the operation's opposite. For instance, to cancel out a 3 that is added to your variable, you subtract 3 from both sides.
- To figure out what to cancel first, second, third,…, follow the reverse order of operations.

Let's look at some examples.

Example 1: Solve the equation 4x – 3 = 21 for x.

We work this problem using the following steps.

$$4x - 3 = 21$$

$$\underline{\quad +3 \quad +3}$$ To cancel out the –3, we add 3 to both sides.

$$4x = 24$$

$$\frac{4x}{4} = \frac{24}{4}$$ To cancel out the 4 multiplied by the x, we divide both sides by 4.

$$x = 6$$

To make sure this is correct, we substitute 6 for x in the original equation:

$$4(6) - 3 \stackrel{?}{=} 21$$

$$24 - 3 \stackrel{?}{=} 21$$

$$21 = 21 \checkmark$$

Since we come up with an obviously true statement, we can conclude that our solution is correct. Also, notice that there is no algebra involved in checking an equation – only basic mathematics such as adding and subtracting.

Example 2: Solve the equation 4x – 7(x + 2) = 2x + 4 for x.

To work this problem, you must remember the Distributive Property:

For all real numbers a, b, and c, a(b + c) = ab + ac and a(b – c) = ab – ac.

There are several ways to work this problem, and the easiest one is shown on the next page.

$4x - 7(x + 2) = 2x + 4$

$4x - 7x - 14 = 2x + 4$ — Use the Distributive Property.

$-3x - 14 = 2x + 4$ — Since the 4x and 7x are on the same side, and since they are like terms, we can combine them.

$\underline{+3x \qquad\quad +3x}$ — To cancel out the –3x, add 3x to both sides.

$-14 = 5x + 4$

$\underline{-4 \qquad\quad -4}$ — To cancel out the + 4, subtract 4 from both sides.

$-18 = 5x$

$\frac{-18}{5} = \frac{5x}{5}$ — To cancel out the multiplication by 5, divide both sides by 5.

$-\frac{18}{5} = x$

Next, we must check this answer by substituting it into the original equation and seeing if we get an obviously true statement (again, without doing any Algebra):

$$4\left(-\frac{18}{5}\right) - 7\left(-\frac{18}{5} + 2\right) \stackrel{?}{=} 2\left(-\frac{18}{5}\right) + 4$$

$$-\frac{72}{5} - 7\left(-\frac{8}{5}\right) \stackrel{?}{=} -\frac{36}{5} + 4$$

$$-\frac{72}{5} + \frac{56}{5} \stackrel{?}{=} -\frac{16}{5}$$

$$-\frac{16}{5} = -\frac{16}{5} \checkmark$$

Example 3: Solve the equation 2x + 3y = 5y – 3 for y.

This question asks us to rearrange the equation so that all the y's are on one side of the equation, and everything else is on the other side. There are several correct ways to work this problem, and one of them is shown below.

$2x + 3y = 5y - 3$

$\underline{\quad -3y \quad -3y}$ — To cancel out the + 3y, we subtract 3y from both sides.

$2x = 2y - 3$

$\underline{\quad +3 \qquad\quad +3}$ — To cancel out the – 3, we add 3 to both sides.

$2x + 3 = 2y$

$\frac{2x+3}{2} = \frac{2y}{2}$ — To cancel out the multiplication by 2, we divide both sides by 2.

$\frac{2x+3}{2} = y$

(Incidentally, you may have worked this problem differently. The solution $y = \frac{-2x-3}{-2}$ is also correct.) Also, many people will try to cancel the 2's.

However, you cannot do this, and the reason for this will be discussed in Chapter 12.

Finally, we must check our answer by substituting $\frac{2x+3}{2}$ for y in our original equation. This time, we must do some Algebra to check our solution, but we will do as little as possible. For example, we will not add or subtract anything from both sides.

$$2x + 3\left(\frac{2x+3}{2}\right) \stackrel{?}{=} 5\left(\frac{2x+3}{2}\right) - 3$$

$$2x + \frac{6x+9}{2} \stackrel{?}{=} \frac{10x+15}{2} - 3$$

Recall that 3 = $\frac{3}{1}$ and 5 = $\frac{5}{1}$. Also, to multiply fractions, you multiply straight across.

$$\frac{4x}{2} + \frac{6x+9}{2} \stackrel{?}{=} \frac{10x+15}{2} - \frac{6}{2}$$

Recall that, to add or subtract fractions, you must have a common denominator.

$$\frac{10x+9}{2} = \frac{10x+9}{2} \checkmark$$

Example 4: Solve the equation $\frac{2a+3b}{5}$ = 7 for a.

This question is asking us to get all the a's on one side and everything else on the other side. To work this problem, you must get rid of the 5 first. This is because the problem actually says (2a + 3b) ÷ 5 = 7, and, when you think about doing the reverse order of operations, you realize that you need to get rid of the 5 before you can do anything with the expression inside the parentheses.

$$\frac{2a+3b}{5} = 7$$

$$\frac{2a+3b}{5} \cdot 5 = 7 \cdot 5$$

To cancel out the division by 5, we multiply both sides by 5.

$$2a + 3b = 35$$

$$\underline{\quad - 3b \quad - 3b}$$

To cancel out the $+3b$, we subtract 3b from both sides.

$$2a = 35 - 3b$$

$$\frac{2a}{2} = \frac{35-3b}{2}$$

To cancel out the multiplication by 2, we divide both sides by 2.

$$a = \frac{35-3b}{2}$$

It is possible to check this solution, but this is a good bit more complicated than the last three examples. Therefore, we will not check this example.

Example 5: Solve the equation $4x + \sqrt{5} = \sqrt{6} - \sqrt{5}$ for x.

To work this problem, we can follow the steps shown on the next page.

$$\begin{aligned} 4x + \sqrt{5} &= \sqrt{6} - \sqrt{5} \\ -\sqrt{5} \quad & \quad\quad -\sqrt{5} \end{aligned}$$

To cancel out the $+\sqrt{5}$, we subtract $\sqrt{5}$ from both sides.

$$4x = \sqrt{6} - 2\sqrt{5}$$

Recall that adding and subtracting roots is just like adding and subtracting like terms. For instance, $-\sqrt{5} - \sqrt{5} = -1\sqrt{5} - 1\sqrt{5} = -2\sqrt{5}$.

$$\frac{4x}{4} = \frac{\sqrt{6} - 2\sqrt{5}}{4}$$

To cancel out the multiplication by 4, we divide both sides by 4.

$$x = \frac{\sqrt{6} - 2\sqrt{5}}{4}$$

Next, we check this problem as follows:

$$4\left(\frac{\sqrt{6} - 2\sqrt{5}}{4}\right) + \sqrt{5} \stackrel{?}{=} \sqrt{6} - \sqrt{5}$$

$$\sqrt{6} - 2\sqrt{5} + \sqrt{5} \stackrel{?}{=} \sqrt{6} - \sqrt{5}$$

$$\sqrt{6} - \sqrt{5} = \sqrt{6} - \sqrt{5} \quad \checkmark$$

Again, recall that we add and subtract roots like we add and subtract like terms.

Problems – Solve each of the following equations for the indicated variable.

1. $4x + 7 = -17$ $x =$ __________

2. $7y - 8 = -8$ $y =$ __________

3. $3 - 4a = 7$ $a =$ __________

4. $20 - 3k = k$ $k =$ __________

5. $6n - 17 = -9$ $n =$ __________

6. $3y + 16 = -5$ $y =$ __________

7. $5p - 2(p + 1) = 7$ $p =$ __________

8. $b - 3(b + c) = 4c$ $b =$ __________

9. $3x + 4y = 5$ $y =$ __________

10. $7x^2 - 2y = 3x$ $y =$ __________

11. $\frac{x}{5} + 3 = 7$ x = __________

14. $\frac{2y+1}{5} = 3x$ y = __________

12. $\frac{y}{x} + 7 = 4$ y = __________

15. $3ab + 2b^2 = 5$ a = __________

13. $\frac{7x-2}{3} = 4$ x = __________

16. $7\sqrt{3} + 2q = 4\sqrt{3}$ q = __________

17. $4a\sqrt{7} + 3x = 5$ x = _____________

18. $8\sqrt{5} - 7(b + \sqrt{5}) = 4c$ b = _____________

19. $3\sqrt{2} + 7(n - 3\sqrt{2}) = 5\sqrt{2}$ n = _____________

20. $4x^2y - 3x = 1$ y = _____________

21. $7m^5 + 4(8k - 3m^5) = -5m^5$ k = _____________

22. $\frac{y-5x}{2a} = 3b$ x = _____________

23. $\frac{2a+3b}{5c} = a$ b = _____________

24. $\frac{x}{3} - 4\sqrt{6} = 3\sqrt{6}$ x = _____________

Part II – Review of More Advanced Equations

Two types of equation that you will see in this section are equations that have no solution and equations that have infinitely many solutions. If you are solving an equation (NOT checking it) and you get an obviously false statement (such as 3 = 8 or 10 = –10), then the equation has no solution. This means that there is no value for your variable that will give you a true sentence when you substitute it in. If you are solving an equation (NOT checking it) and you get an obviously true statement (such as 3 = 3, x = x, or –10 = –10), then the equation has infinitely many solutions. For now, we will say that this means that any value for the variable will work when you substitute it in.

Example 1: Solve the equation $2 + 3(a - 2) = a - 4 + 2a$ for a.

$$2 + 3(a - 2) = a - 4 + 2a$$

$$2 + 3a - 6 = a - 4 + 2a$$

Many students, when they are asked to solve a problem like this, will try to add the 2 and 3 before they do anything else. However, you cannot do this because the order of operations says you must multiply before you add or subtract. Therefore, you must use the Distributive Property first.

$$3a - 4 = 3a - 4$$

Since this is an obviously true equation, we can conclude that this equation has an infinite number of solutions. To check this, we can pick any number and substitute it for a into the original equation (we will choose a = 10).

$$2 + 3(10 - 2) \stackrel{?}{=} 10 - 4 + 2(10)$$

$$2 + 3(8) \stackrel{?}{=} 10 - 4 + 20$$

$$26 = 26 \checkmark$$

Since this gave us an obviously true equation, we can conclude that there are in fact an infinite number of solutions to this problem.

Example 2: Solve the equation $4k + 3k - 2 = 7(k - 1)$ for k.

$$4k + 3k - 2 = 7(k - 1)$$

$$7k - 2 = 7k - 7$$

Since the 4k and the 3k are on the same side of the equation and are like terms, we can combine them. We can also use the Distributive Property to distribute the 7.

$$-2 = -7$$

Subtract 7k from both sides.

Since this is an obviously false statement, we must conclude that the equation has no solution. Also, since there is no solution, we cannot check our answer.

Another type of equation that you will see in this section is a problem with two fractions and an equal sign in the middle. The easiest way to work this type of problem is to use the Cross Multiplication Principle:

If $\frac{a}{b} = \frac{c}{d}$, then (a)(d) = (b)(c).

Example 3: Solve the equation $\frac{k+2}{y} = \frac{x}{25}$ for y.

The easiest way to solve this problem is to follow the steps below.

$25(k + 2) = xy$	Use the Cross Multiplication Principle.
$25k + 50 = xy$	Simplify using the Distributive Property.
$\frac{25k+50}{x} = \frac{xy}{x}$	To cancel out the multiplication by x, we divide both sides by x.
$\frac{25k+50}{x} = y$	

It is possible to check this answer, but it is more complicated than the others, and so we will not worry about checking this answer.

A fourth type of problem that you will see in this section is an equation with absolute value signs in it. Recall that absolute value signs say, "Do whatever is inside of me, and then make it positive." Also, to work many of the problems in this section, you will need to use the following principle.

> Suppose that c represents a number greater than or equal to zero. Then $|x| = c$ (or, equivalently, $c = |x|$) means that $x = c$ or $x = -c$.

Example 4: Solve the equation $|x - 5| + 2 = |7 - 13|$ for x.

To work this problem, we will use the principle above, but we must first rearrange it so that the absolute value is by itself on one side. Also, note that, because the 5 is inside the absolute value signs, we CANNOT cancel it out by adding 5 to both sides. We must get it outside the absolute value signs (using the principle above) first.

$|x - 5| + 2 = |7 - 13|$	
$|x - 5| + 2 = |-6|$	Simplify: $7 - 13 = -6$.
$|x - 5| + 2 = 6$	Simplify further: $|-6| = 6$.
$|x - 5| = 4$	Cancel out the + 2 by subtracting 2 from both sides.
$x - 5 = 4$ or $x - 5 = -4$	Use the principle above.
$x = 9$ or $x = 1$	Add 5 to both sides of each equation.

Next, we must check both of these answers.

$|9 - 5| + 2 \stackrel{?}{=} |7 - 13|$	and	$|1 - 5| + 2 \stackrel{?}{=} |7 - 13|$
$4 + 2 \stackrel{?}{=} 6$	and	$4 + 2 \stackrel{?}{=} 6$
$6 = 6$ ✓	and	$6 = 6$ ✓

This tells us that our solutions of 9 and 1 are both correct. You **MUST** list both of them, or your answer will be counted wrong.

Example 5: Solve the equation |a + 1| + 3 = 2a for a.

To work this problem, we will start by getting |a + 1| by itself on one side of the equation, and then we can use the principle on the previous page again.

$|a + 1| + 3 = 2a$

$|a + 1| = 2a - 3$ Get the absolute value by itself by subtracting 3 from both sides.

$a + 1 = 2a - 3$ or $a + 1 = -(2a - 3)$ Use the principle on the previous page.

$a + 4 = 2a$ or $a + 1 = -2a + 3$ Solve for a.

$4 = a$ or $3a = 2$

$4 = a$ or $a = \frac{2}{3}$

Next, of course, we must check both our solutions.

$|4 + 1| + 3 \stackrel{?}{=} 2(4)$ and $\left|\frac{2}{3}+1\right| + 3 \stackrel{?}{=} 2\left(\frac{2}{3}\right)$

$5 + 3 \stackrel{?}{=} 8$ and $\frac{5}{3} + 3 \stackrel{?}{=} \frac{4}{3}$

$8 = 8$ ✓ and $\frac{14}{3} \neq \frac{4}{3}$ ✗

Since substituting 4 in the original equation gave us a true equation but $\frac{2}{3}$ did not, we can conclude that 4 is a correct answer, but $\frac{2}{3}$ is not. Therefore, either we did something wrong somewhere, or else the second half of the problem has no solution. Since a check of the Algebra leading up to this solution says it is correct, we must assume that 4 is the only correct solution to this equation. (By the way, if neither of the solutions had worked, we would have said that the problem had "no solution.")

Problems – Solve for the variable indicated. (If there is no solution, write "no solution" in the blank. If there are an infinite number of solutions, write "infinite solutions" in the blank.)

25. $\frac{4x+2y}{5} = \frac{1-x}{3}$ x = ______________________

26. $\frac{1-5x}{5} = \frac{a}{y}$ a = ______________________

27. $\frac{3x - 4y}{2} = \frac{12x - 16y}{8}$ y = ____________________

28. $\frac{2a - 3b}{5} = \frac{3}{4a}$ b = ____________________

29. $\frac{4 + x}{y} = k$ y = ____________________

Hint: Recall that $k = \frac{k}{1}$.

30. $\frac{x - y}{a - 3b} = c$ a = ____________________

31. $\frac{x - y}{a - 3b} = c$ b = ____________________

32. $|x| - 5 = 4$ x = ____________________

33. $|p - 1| = 3$ p = ____________________

34. $a - 1 = |7|$ a = ____________________

35. $\frac{x + 1}{y} = \frac{x - 5}{y}$ x = ____________________

36. $3 + 2|k - 5| = 9$ k = ____________________

37. $|10 - 6| - 2d = |d|$ d = ____________________

38. $|3n - 2| + 5 = 1$ n = ______________________

39. $3 - 5(x + 2) = -5x$ x = ______________________

40. $|2a - 3| = a$ a = ______________________

41. $|a - 5| = a + 11$ a = ______________________

42. $|x - 1| + 5 = 2x$ x = ______________________

43. $\frac{|n + 6|}{3} = 4$ n = ______________________

44. $\frac{y}{|y + 3|} = \frac{1}{2}$ y = ______________________

45. $\frac{y}{|y - 2|} = \frac{1}{5}$ y = ______________________

46. $\frac{3}{5}x = \frac{3}{5}x + 6$ x = ______________________

47. $4 - 5|q - 4| = 14$ q = ______________________

48. Challenge: $x - 1 = |x - 1|$ x = ______________________

Part III – Review of Solving Inequalities

An <u>inequality</u>, as you may recall, is just like an equation except it has a <, >, ≤, ≥, or ≠ sign in the middle instead of an equal sign. Inequalities are solved just like equations – you can do almost anything you want to one side of the equation as long as you do exactly the same thing to the other side. The only exception is that, when you multiply or divide both sides by a negative number, you must flip the inequality sign.

Let's look at some examples.

Example 1: Solve for x: $4x - 1 < -17$

$$4x - 1 < -17$$

$$\underline{\quad +1 \quad +1}$$

To cancel out the –1, add 1 to both sides.

$$4x < -16$$

$$\frac{4x}{4} < \frac{-16}{4}$$

To cancel out the multiplication by 4, divide both sides by 4. (Note that we divided both sides by +4, and so we did not flip the inequality sign.)

$$x < -4$$

There are actually two parts to checking an inequality: you must check the number, and you must check the direction of the inequality sign. To check the number, you substitute the number in for x in the original inequality, and you make sure that both sides **equal** each other. To check the inequality sign, pick a number that actually falls in the range of your solution (in this case, we want a number less than -4), and substitute that in your original inequality and make sure you get a true statement.

To check the number:

$$4(-4) - 1 \stackrel{?}{=} -17$$

$$-16 - 1 \stackrel{?}{=} -17$$

$$-17 = -17 \checkmark$$

To check the inequality sign:
(We picked x = –5.)

$$4(-5) - 1 \stackrel{?}{<} -17$$

$$-20 - 1 \stackrel{?}{<} -17$$

$$-21 < -17 \checkmark$$

Since both parts checked, we can assume that our solution of $x < -4$ is correct.

Example 2: Solve for y: $3x - 2y \geq 5$ for y.

$$3x - 2y \geq 5$$

$$\underline{-3x \qquad -3x}$$

$$-2y \geq 5 - 3x$$

To cancel out the 3x, subtract 3x from both sides. (Many people try to *add* 3x here because addition is the opposite of subtraction. However, this is not correct because the subtraction sign goes with the 2y, and not the 3x. Also note that, if you add 3x to both sides, you get 6x, and not zero, on the left side.)

$$\frac{-2y}{-2} \not\geq \leq \frac{5 - 3x}{-2}$$

To cancel out the multiplication by –2, divide both sides by –2. (Note that we divided both sides by a negative number, and so we had to flip the inequality sign.)

$$y \leq \frac{5 - 3x}{-2}$$

Checking inequalities with more than one variable can be tricky. We won't worry about checking the direction of the inequality sign, but you can check the numbers just like we did earlier:

$$3x - 2\left(\frac{5-3x}{-2}\right) \overset{?}{=} 5$$

$$3x + 5 - 3x \overset{?}{=} 5$$

Note that the multiplication by –2 and the division by –2 cancel each other out.

$$5 = 5 \checkmark$$

Example 3: Solve for q: $8 - \frac{q}{7} \geq 5$

$$8 - \frac{q}{7} \geq 5$$

$$-\frac{q}{7} \geq -3$$

Subtract 8 from both sides.

$$-\frac{q}{7} \cdot -7 \overset{\not\geq}{\leq} -3 \cdot -7$$

To cancel out division by –7, we multiply both sides by –7. Note that we must flip the inequality sign because we multiplied both sides by a negative number.

$$q \leq 21$$

Next, of course, we must check this solution. Recall that we need to substitute 21 for q in the original inequality and make sure that the two sides **equal** each other, and we need to check the inequality sign by substituting a number in the range of our solution (in this case, we need to pick a number less than or equal to 21.)

To check the number:

$$8 - \frac{21}{7} \overset{?}{=} 5$$

$$8 - 3 \overset{?}{=} 5$$

$$5 = 5 \checkmark$$

To check the inequality symbol:
(We picked q = 7.)

$$8 - \frac{7}{7} \overset{?}{\geq} 5$$

$$8 - 1 \overset{?}{\geq} 5$$

$$7 \geq 5 \checkmark$$

Example 4: Solve for b: $1 - 3b \neq 5$

This problem is solved exactly like an equation – there's just a ≠ sign in the middle.

$$1 - 3b \neq 5$$

$$-3b \neq 4$$

Subtract 1 from both sides.

$$b \neq -\frac{4}{3}$$

Divide both sides by –3. (Technically, we should flip the inequality, but flipping it just gives us the same thing.)

To check this solution, we do not need to check the direction of the inequality symbol (because there is no direction). We do, however, need to substitute our answer of $-\frac{4}{3}$ in and make sure that both sides equal each other:

$$1 - 3\left(-\frac{4}{3}\right) \stackrel{?}{=} 5$$

$$1 \quad + \quad 4 \quad \stackrel{?}{=} 5$$

$$5 \quad = 5 \quad \checkmark$$

Example 5: Solve for p: px + 5 < 7y

This problem starts out just like the others did, but then it gets a little harder at the end, as you will see.

$px + 5 < 7y$

$px < 7y - 5$ Subtract 5 from both sides.

Now, at this point, we need to divide both sides by x, but we don't know if we need to flip the inequality sign or not (because we don't know if x is positive or negative). So, we will state two cases:

If x > 0:

$$\frac{px}{x} < \frac{7y-5}{x}$$

$$p < \frac{7y-5}{x}$$

If x < 0:

$$\frac{px}{x} \not< \; > \frac{7y-5}{x}$$

$$p > \frac{7y-5}{x}$$

Therefore, our final answer is $p < \frac{7y-5}{x}$ (if x > 0) or $p > \frac{7y-5}{x}$ (if x < 0). Note that we do not need to worry about what happens if x = 0 because the denominator of a fraction can never equal zero.

Example 6: Solve for y: ax + by ≤ c

This problem is worked just like the one in Example 5.

$ax + by \le c$

$by \le c - ax$ Subtract ax from both sides.

$\frac{by}{b} \le \frac{c-ax}{b}$ (if b > 0) or $\frac{by}{b} \not\le \; \ge \frac{c-ax}{b}$ (if b < 0)

$y \le \frac{c-ax}{b}$ (if b > 0) or $y \ge \frac{c-ax}{b}$ (if b < 0)

Again, notice that we do not worry about what happens if b = 0 because that would make the fraction undefined.

Example 7: Solve for n: 7 – m(n + 1) > 3m

This problem is just like the last two, with one exception: at the end, when you need to divide both sides by –m, you need to realize that –m is actually positive when m **>** 0 (and vice versa).

$7 - m(n + 1) > 3m$

$7 - mn - m > 3m$ Use the Distributive Property to simplify.

$-mn > 4m - 7$ Add m to both sides and subtract 7 from both sides.

$\frac{-mn}{-m} \underset{<}{\not>} \frac{4m-7}{-m}$ (if m > 0) or $\frac{-mn}{-m} > \frac{4m-7}{-m}$ (if m < 0)

$n < \frac{4m-7}{-m}$ (if m > 0) or $n > \frac{4m-7}{-m}$ (if m < 0)

Problems – Solve for the indicated variable. (Do not forget to include the inequality sign in your answer!)

49. Solve for x: $3x + 1 < 16$ ________________

50. Solve for y: $7y - 5 \geq 17$ ________________

51. Solve for n: $3n - (5n + 1) \leq 8n$ ________________

52. Solve for a: $\frac{a}{5} + 8 > -3$ ________________

53. Solve for k: $5 - \frac{k}{3} \leq 7$ ________________

54. Solve for c: $3 + 2(c - 1) \neq 5c$ ________________

55. Solve for m: $4m - 5(m + y) \leq 2y$ ________________

56. Solve for q: $\frac{q}{5} - 3 > \frac{1}{5}(2 + q)$ ________________

57. Solve for x: $7 - 3x \leq -5$ ________________

58. Solve for b: $3a - 2(b - a) < 3$ ________________

59. Solve for x: $18 - 5(x + 1) > 2x - 7x$ ________________

60. Solve for p: $8 > 3p + 4(k - 2p)$ ________________

61. Solve for b: $4a + 2a(b + 3) \geq -1$ ________________

62. Solve for y: $3w + 2x(y + 1) \leq 5x$ ________________

63. Solve for a: $14 > 2b + 3ac$ ________________

64. Solve for d: $6d - 7c(a - b) < 4ac$ ________________

65. Solve for p: $3p + 5(p - 1) \leq 8p + 4$ ________________

66. Solve for x: $2a + 1 > 2a + 3x$ ________________

67. Solve for n: $4a + a(2 - 3n) \geq 7$ ________________

68. Solve for y: $x - \frac{2y}{3} \neq 5x$ ________________

69. Solve for k: $4 - 5a(2k + 1) < -2a$ ________________

70. Solve for c: $\frac{c}{a} \leq 3$ ________________

71. Solve for w: $7 > 5 - \frac{3w}{4}$ ________________

72. Solve for x: $2x + 1 \geq 3x - x$ ________________

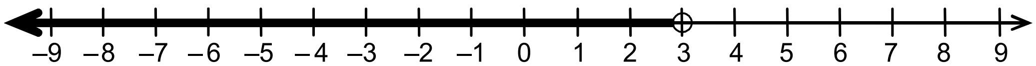
73. Solve for y: $1 - \frac{y}{x} < -6$ ________________

74. Solve for b: $a + \frac{b}{c} > 5a$ ________________

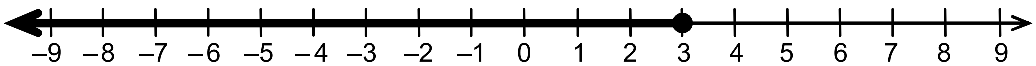
75. Solve for m: $4 \geq 4 - a\left(\frac{m}{3} - 5\right)$ ________________

Part IV – Graphing Inequalities on Number Lines

You may recall that an inequality that has only one variable can be graphed on a number line. For example, to graph "$x < 3$," we need to show all the numbers that are less than 3. We do this by putting an open circle around the 3 (the open circle says that x cannot equal 3) and then shading the part of the number line that shows the numbers less than 3, as shown below.



If we had wanted to graph "$x \le 3$," then we would have filled in the circle around the 3, as shown below.



Now, let's look at some more complicated examples.

Example 1: Graph on a number line: $2x - 1 > 7$

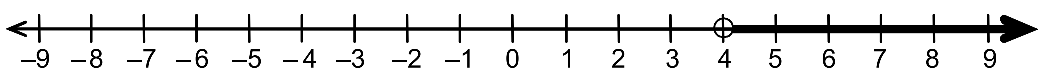
Before we can graph the solution to this inequality, we must first solve for x:

$$2x - 1 > 7$$

$$2x > 8 \qquad \text{Add 1 to both sides.}$$

$$x > 4 \qquad \text{Divide both sides by 2.}$$

Now we can graph this solution on a number line. Note that x cannot equal 4, and so we need an open circle around the 4. Also note that we want to talk about the numbers that are bigger than (ie, to the right of) 4. Therefore, our graph looks like this:



Example 2: Graph on a number line: $p + 2(p - 1) \neq 3p + 2$

Once again, we must start by solving for p:

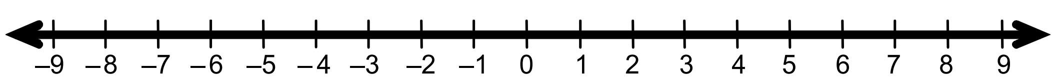
$$p + 2(p - 1) \neq 3p + 2$$

$$p + 2p - 2 \neq 3p + 2 \qquad \text{Simplify using the Distributive Property.}$$

$$3p - 2 \neq 3p + 2 \qquad \text{Combine like terms.}$$

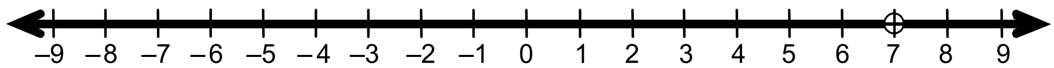
$$-2 \neq 2 \qquad \text{Subtract 3p from both sides.}$$

This is an obviously true statement, and so we must conclude that there are an infinite number of solutions. As we said earlier in this chapter, this means that any number you choose to substitute in for p will work. Hence, to graph the solution on a number line, the entire number line must be shaded:



Example 3: Graph on a number line: $y \neq 7$

In this case, the variable is already by itself, and so all we have to do is graph it. The statement "$y \neq 7$" means that y can be anything bigger than 7 or smaller than 7, but it cannot equal 7. So, we put an open circle around the 7 and shade the rest of the number line, as shown below.



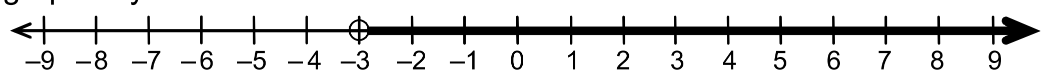
The next several examples discuss conjunctions and disjunctions. <u>Conjunctions</u> are two statements with the word "and" in the middle (for example, "$x > 4$ and $x < 7$"), and <u>disjunctions</u> are two statements with an "or" in the middle (for example, "$x > 4$ or $x < 7$"). The word "and" says that you need to make both inequalities true at the same time. The word "or" says to talk about where either one (but not necessarily both) is true.

Example 4: Graph on a number line: $y - 4 > -7$ and $3 - y \geq 1$

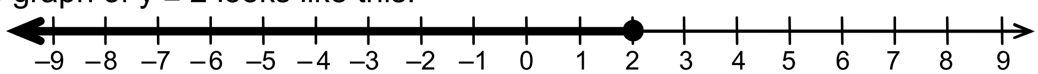
We must begin by solving each of these inequalities for y:

$y - 4 > -7$			and	$3 - y \geq 1$	
y	> -3	Add 4 to both sides.	and	$-y \geq -2$	Subtract 3 from both sides.
y	> -3		and	$y \underset{\leq}{\not\geq} 2$	Divide both sides by –1.

Next, we must graph $y > -3$ and $y \leq 2$. To do this, we first need to graph $y > -3$ and $y \leq 2$ separately, and then our final answer will be wherever the two overlap. The graph of $y > -3$ looks like this:



The graph of $y \leq 2$ looks like this:



Therefore, the two graphs overlap between –3 and 2, including 2 but not including –3. The final answer looks like this:

Example 5: Graph on a number line: $7 \geq k$ and $2k - 1 < 5$

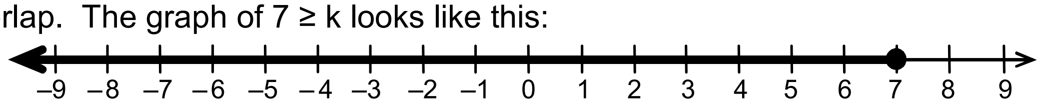
We begin by solving for k in the second inequality (in the first inequality, k is already by itself, and so we do not need to worry about it):

$2k - 1 < 5$

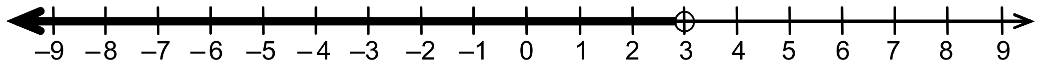
$2k < 6$

$k < 3$

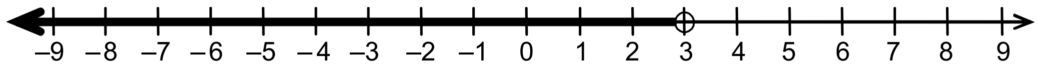
So, now we need to graph $7 \geq k$ and $k < 3$ separately and then see where they overlap. The graph of $7 \geq k$ looks like this:



The graph of $k < 3$ looks like this:



Therefore, the final answer looks like this:



Example 6: Graph on a number line: $4 - \frac{x}{3} \le 5$ or $x > 7$

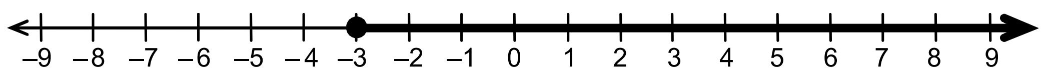
We must, of course, begin by solving the first inequality for x:

$4 - \frac{x}{3} \le 5$

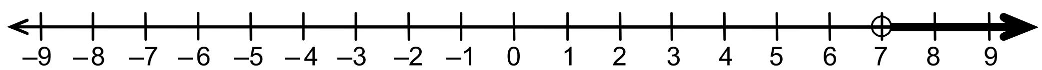
$-\frac{x}{3} \le 1$ Subtract 4 from both sides.

$x \ge -3$ Multiply both sides by –3, and don't forget to flip the inequality sign.

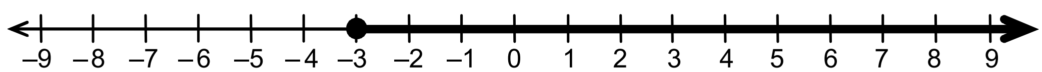
Now we need to graph $x \ge -3$ or $x > 7$. Once again, we will graph both of them separately and then figure out the final answer from there. The graph of $x \ge -3$ looks like this:



The graph of $x > 7$ looks like this:



When you remember that we said the word "or" says to talk about where either one (but not necessarily both) is true, you should realize that the final answer looks like this:

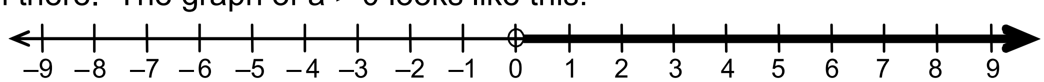


Example 7: Graph on a number line: $a > -3a$ or $2 - a > a$

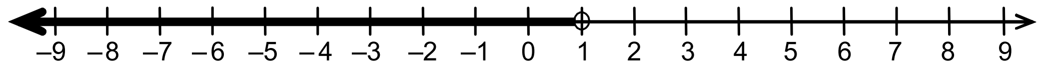
We begin by solving each inequality for a:

$a > -3a$		or	$2 - a > a$		
$4a > 0$	Add 3a to both sides.	or	$2 > 2a$		Add a to both sides.
$a > 0$	Divide both sides by 4.	or	$1 > a$		Divide both sides by 2.

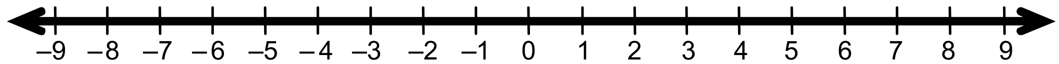
Now, we need to graph $a > 0$ and $1 > a$ separately and figure out the final answer from there. The graph of $a > 0$ looks like this:



The graph of 1 > a looks like this:



This tells us that the final answer is the entire line shaded (because at least one of these inequalities is true for every number on the number line):

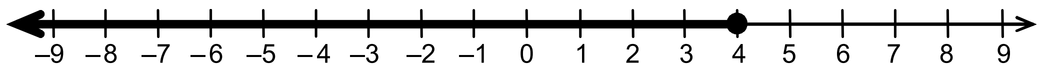


Example 8: Graph on a number line: $d - 3(d + 1) \geq -11$ or $d + 1 \leq 3d - 14$

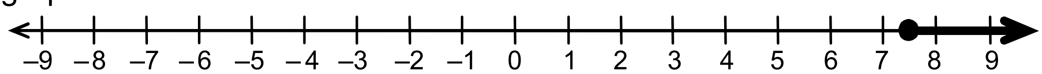
We start by solving each inequality for d:

$d - 3(d + 1) \geq -11$	or	$d + 1 \leq 3d - 14$
$d - 3d - 3 \geq -11$	or	$-2d + 1 \leq -14$
$-2d - 3 \geq -11$	or	$-2d \leq -15$
$-2d \geq -8$	or	$d \geq \frac{15}{2}$
$d \leq 4$	or	$d \geq 7.5$

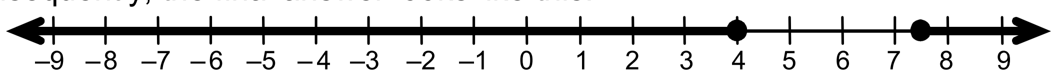
Now, we can graph $d \leq 4$ or $d \geq 7.5$. The graph of $d \leq 4$ looks like this:



The graph of $d \geq 7.5$ looks like this:

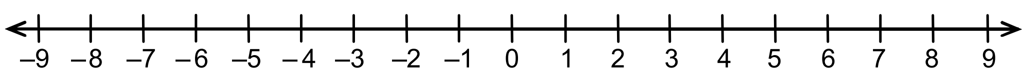


Consequently, the final answer looks like this:

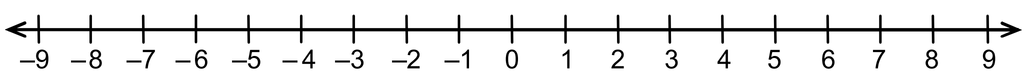


Problems – Graph each of the following on a number line.

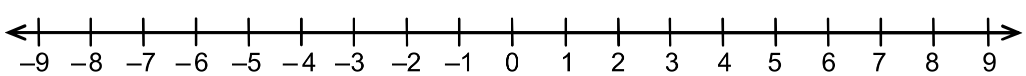
76. $x \leq -1$



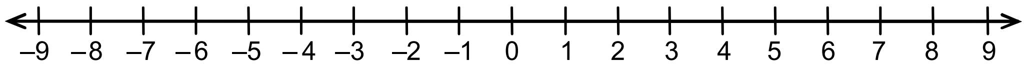
77. $y > 5$



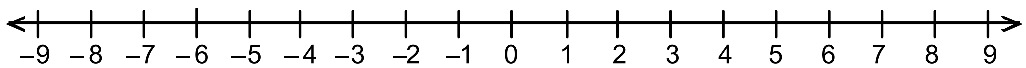
78. $3x - 5 < 10$



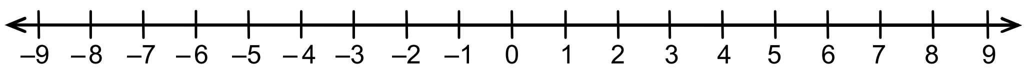
79. $4c + 1 \geq 9$



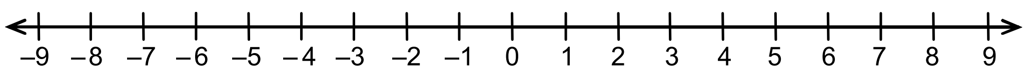
80. $3x – (x + 1) \neq 4$



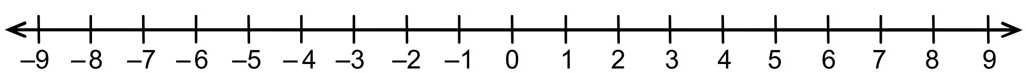
81. $\frac{1}{x} < 1$ (**Hint: Be careful about the direction of the inequality sign!)**



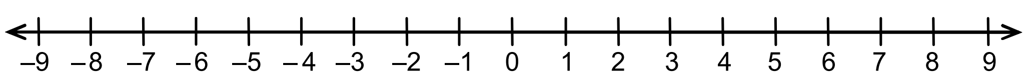
82. $\frac{-3}{x} \geq 1$



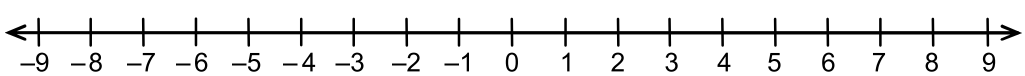
83. $5 – 2x < 3 + 2(x + 2)$



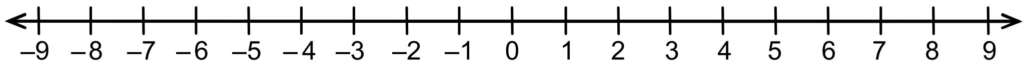
84. $4y + 3 > y – 3(y + 1)$



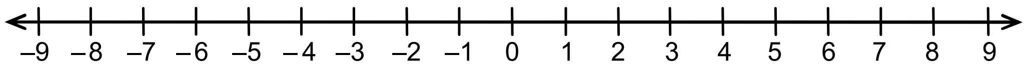
85. $5n – 5(n + 2) \neq 5$



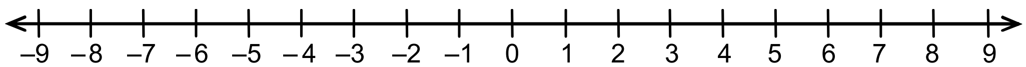
86. $k < 5$ and $k > -1$



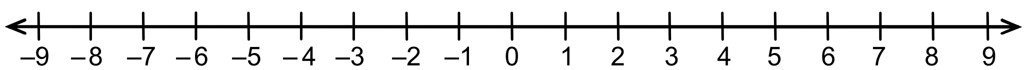
87. $m + 1 \ge 5$ and $m - 1 > 5$



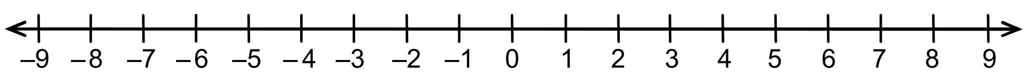
88. $y - 5 < 4$ or $y \ge 7$



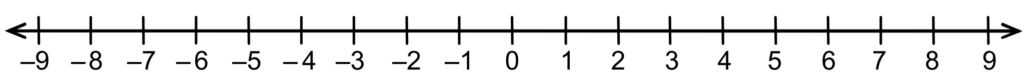
89. $c + 3 > 2c$ or $c + 5 < 4$



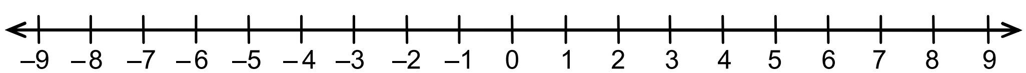
90. $x + 1 \ne 5$ and $x - 3 > 4x$



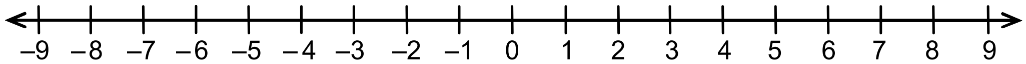
91. $y - \frac{1}{3}y < 5$ or $y \ne 3$



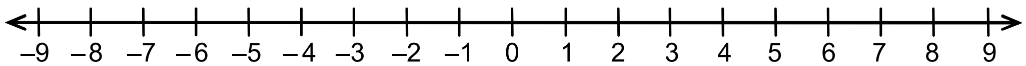
92. $m - 4(m - 3) < 2m$ and $2m \ge 7$



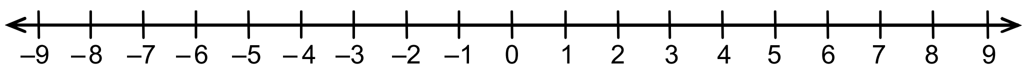
93. $\frac{t}{2} \geq -1$ and $1 - \frac{3t}{2} > -2$



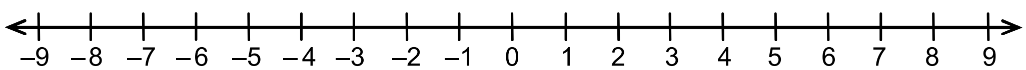
94. $\frac{4x+5}{3} < 2$ or $\frac{7x-1}{-2} \leq -10$



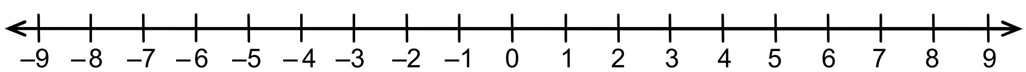
95. $x + 3 > 5$ and $x + 3 < 5$



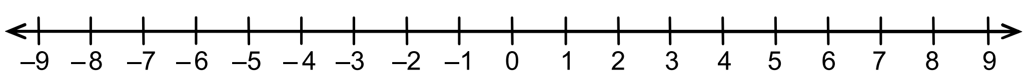
96. $x > 5$ or $3x - 1 < 5$



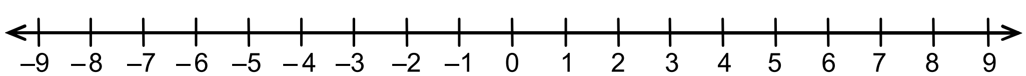
97. $n \geq 5n + 1$ or $3 + n < 5 - n$



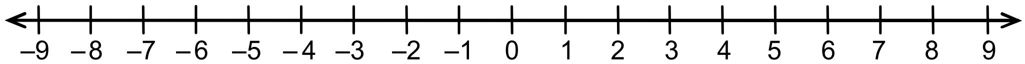
98. $3m - 1 \neq 7$ or $2m + \frac{1}{2} < 3$



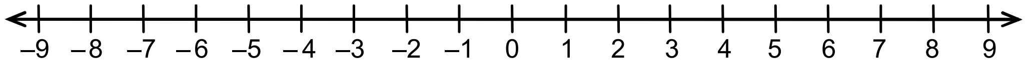
99. $4p + 3 > 4p - 3$ and $3(p - 1) \geq 5$



100. $x + 1 < x - 1$ or $x \geq 3$

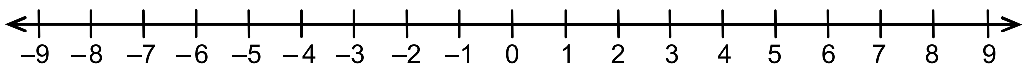


101. $2 - (x + 1) \leq 5 - x$ and $4 - 3x < 7$

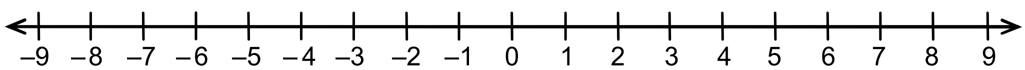


The next few problems are in the form $a < x < b$ (where $a < b$). The statement "$a < x < b$" means, "$a < x$ and $x < b$." For example, "$1 < x + 3 \leq 5$" means, "$1 < x + 3$ and $x + 3 \leq 5$."

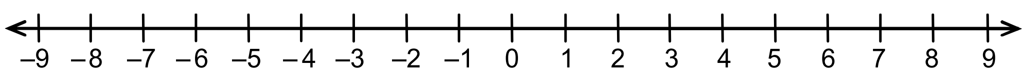
102. $3 \leq x < 5$



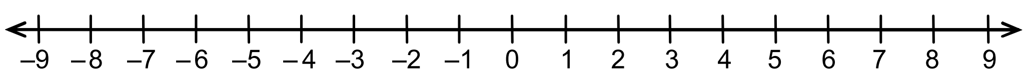
103. $-1 < x - 1 < 5$



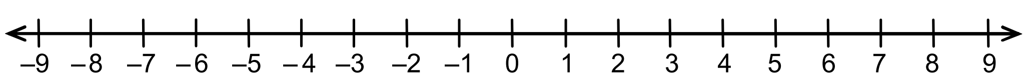
104. $-1 < \frac{n}{2} \leq 3$



105. $2 \leq 1 - 3a \leq 7$



106. $-1 < 5 - \frac{2y}{3} \leq 3$



Part V – Inequalities Involving Absolute Value

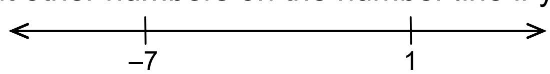
In this section, we will discuss how to solve inequalities involving absolute value symbols and how to graph the solution on a number line. You will see variations of the methods we use in this section throughout this book in the sections involving inequalities.

Example 1: Solve |x + 3| < 4. Represent your answer both with a number line and symbolically.

Step 1: Pretend that there is really an equal sign in the middle and solve the resulting equation.

$$|x + 3| = 4$$
$$x + 3 = 4 \text{ or } x + 3 = -4$$
$$x = 1 \text{ or } x = -7$$

Step 2: Draw a number line, and put the numbers you found in Step 1 on it. You can put other numbers on the number line if you want to, but you do not have to.



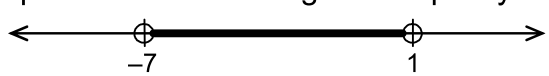
Step 3: Test the different regions on the number line by picking numbers in each region and substituting them into the original inequality to see if you get a true statement or a false statement.

For this example, we have three different regions we need to test: the numbers less than –7, the numbers between –7 and 1, and the numbers more than 1.

Test a number less than –7:	Test a number between –7 and 1:	Test a number more than 1:
(We chose x = –8.)	(We chose x = 0.)	(We chose x = 5.)
$\lvert -8 + 3\rvert \overset{?}{<} 4$	$\lvert 0 + 3\rvert \overset{?}{<} 4$	$\lvert 5 + 3\rvert \overset{?}{<} 4$
$\lvert -5\rvert \overset{?}{<} 4$	$\lvert 3\rvert \overset{?}{<} 4$	$\lvert 8\rvert \overset{?}{<} 4$
$5 \nless 4$ ✗	$3 < 4$ ✓	$8 \nless 4$ ✗

Step 4: Graph the solution on a number line.

Note that the inequality turned out to be true when we tested a number between –7 and 1, but it was false when we tested the other numbers. Therefore, we will shade the numbers between –7 and 1. Also, the circles will be open because the original inequality was <, not ≤ or ≥.



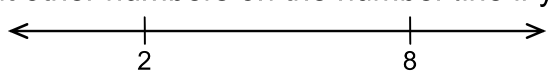
Now, to represent our answer symbolically, you need to remember what we discussed in the last section. We can write "x > –7 and x < 1" or, equivalently, "–7 < x < 1."

Example 2: Solve the inequality $7 - 2|5 - y| \le 1$. Represent your answer both with a number line and symbolically.

Step 1: Pretend that there is really an equal sign in the middle and solve the resulting equation.

$$7 - 2|5 - y| = 1$$
$$-2|5 - y| = -6$$
$$|5 - y| = 3$$
$$5 - y = 3 \text{ or } 5 - y = -3$$
$$y = 2 \text{ or } y = 8$$

Step 2: Draw a number line, and put the numbers you found in Step 1 on it. You can put other numbers on the number line if you want to, but you do not have to.



Step 3: Test the different regions on the number line by picking numbers in each region and substituting them into the original inequality to see if you get a true statement or a false statement.

We need to test three regions: the numbers less than 2, the numbers between 2 and 8, and the numbers more than 8.

Test a number less than 2:	Test a number between 2 and 8:	Test a number more than 8:
(We chose $y = 0$.)	(We chose $y = 3$.)	(We chose $y = 9$.)
$7 - 2\|5 - 0\| \overset{?}{\le} 1$	$7 - 2\|5 - 3\| \overset{?}{\le} 1$	$7 - 2\|5 - 9\| \overset{?}{\le} 1$
$7 - 2\ \|5\| \overset{?}{\le} 1$	$7 - 2\ \|2\| \overset{?}{\le} 1$	$7 - 2\ \|-4\| \overset{?}{\le} 1$
$7 - 10 \overset{?}{\le} 1$	$7 - 4 \overset{?}{\le} 1$	$7 - 8 \overset{?}{\le} 1$
$-3 \le 1$ ✓	$3 \not\le 1$ ✗	$-1 \le 1$ ✓

Step 4: Graph the solution on a number line.

Note that the inequality was true when we tested a number less than 2 and when we tested a number more than 8. Thus, we shade the numbers that are less than 2 and the numbers that are more than 8. Also, since the original inequality was ≤, and not < or >, the circles will be closed.

Now, if you remember what we discussed in the last section, you should realize that you can represent this solution symbolically by saying "$y \le 2$ or $y \ge 8$."

Problems – Solve each of the following inequalities. Represent your final solutions both symbolically and with a number line.

107. $|x| < 5$

108. $|x| \ge 3$

109. $|y - 3| \geq 4$

110. $|b + 1| \leq 2$

111. $|2 - a| > 5$

112. $4 + |n - 1| > 7$

113. $3 > |c - 2|$

114. $3|y| - 2 > 7$

115. $7 - |5m + 3| > 7$

116. $|x - 3| + 2 \geq 2$

117. $7 > |2x + 5| + 2$

118. $4 - |3w + 1| \leq 6$

119. $2|5a + 3| + 1 < 7$

120. $9 - 2|5x - 6| < 1$

121. $|3p + 5| < -2$

122. $8 + 2|y + 3| < 8$

123. $5 - 2|3d + 1| \leq 1$

124. $3 + 2|m| > 5$

125. $4 + |5k - 3| \geq 1$

126. $5 - |5x - 1| > 3$

127. $2 - 3|y - 6| < 4$

128. $5 - 3|n| \geq 4$

129. $5 \leq 3 - |2p|$

130. $|3v + 4| < 5$

131. $5 - |4 - 7q| < 2$

132. $3 - |4x| < 5$